UG-00-08

# Critical fields on the M5–brane and noncommutative open strings

E. Bergshoeff*, D. S. Berman†, J. P. van der Schaar‡ and P. Sundell§

*Institute for Theoretical Physics, University of Groningen,*

*Nijenborgh 4, 9747 AG Groningen, The Netherlands*

## Abstract

The M5–brane is investigated near critical field-strength. We show that this limit on the M5–brane reduces to the noncommutative open string limit on the D4–brane. The reduction on a two–torus leads to both the noncommutative open string limit and the noncommutative Yang–Mills limit on the D3–brane. The decoupled noncommutative five–brane is identified with the strong coupling limit of the noncommutative open string theory on the D4–brane and S–duality on the noncommutative D3–brane is identified with a modular transformation on the five–brane. We argue that the open membrane metric defines a finite length scale on the worldvolume of the M5–brane in the decoupling limit. This length scale can be associated to the effective length scale of an open membrane.

---

*e.bergshoeff@phys.rug.nl

†d.berman@phys.rug.nl

‡schaar@phys.rug.nl

§p.sundell@phys.rug.nl

## I. INTRODUCTION

Noncommutative geometry has been shown to play a fascinating role in string theory. The original interest was sparked by the appearance of noncommutativity on D-branes in the presence of constant background Neveu–Schwarz two–form potentials $B_{\rm NS}$ [1–3]. On the D–brane itself these potentials appear as a two–form adapted field strength $\mathcal{F} = dA + B_{NS}$. In the magnetic case (i.e. when $\mathcal{F}$ has only spatial parts) a spatially noncommutative Yang–Mills theory (NCYM) on the D–brane can be decoupled from the bulk gravity [3]. The natural, though conceptually challenging generalisation, was to include field strengths with non-zero electric components thus inducing spatio-temporal noncommutativity on the D-brane. This was examined in [4–8]. The somewhat remarkable result was that by examining the D-brane in a decoupling limit near critical electric field strength[1] one naturally constructed a unitary decoupled spatio-temporally noncommutative open string theory (NCOS).

The crucial property of the NCOS limit described in [4,5] is to keep fixed both the open string two–point function and the effective open string coupling constant

$$< X^A X^B >= 2\pi\alpha' G^{AB} + \Theta^{AB} = 2\pi\alpha' \left( \frac{1}{g + 2\pi\alpha'\mathcal{F}} \right)^{AB} , \tag{1}$$

$$G_0 = g_s \frac{det^{1/2}(g + 2\pi\alpha'\mathcal{F})}{det^{1/2}(g)} , \tag{2}$$

where $G^{AB}$ is the symmetric part and $\Theta^{AB}$ the antisymmetric part of the two–point function. In this limit the leading divergent parts of $\mathcal{F}$ cancel the contribution from $g_{\mu\nu}/\alpha'$, leaving the two–point function and coupling governed by the finite subleading terms[2]. As discussed in [4], This NCOS limit introduces a fixed metric $G_{\rm OS}^{AB}$ and a new effective length scale $\alpha'_{\rm eff}$ defined as follows:

[1] The concept of a critical field only works for the electric case as it relies crucially on the Lorentz signature.

[2] Here and in the rest of this paper we will ignore factors of $2\pi$ etc.

$$\alpha' G^{AB} = \alpha'_{\rm eff} G^{AB}_{\rm OS} \ . \tag{3}$$

In this paper we will consider a decoupling limit in the M5–brane analogous to the noncommutative open string limit and examine its properties through an open membrane probe. Even though the analog of the string two–point function is not available for the open membrane, it was conjectured in [9] that the decoupled five–brane theory should be formulated in terms of a so called *open membrane metric* with properties analogous to those of the open string metric. Thus we demand that the six–dimensional proper lengths measured by the open membrane metric are fixed in units of the 11D Planck length $\ell_p$. This defines a finite effective length scale of the decoupled five–brane theory that we shall denote $\ell_g$. This is suggestive of an open membrane theory underlying the decoupled spatio-temporal noncommutative M5–brane.

Evidence for the decoupled non–commutative five–brane theory can be obtained by comparing various reductions of the five–brane limit to NCOS and NCYM limits in string theory. Conversely, this provides a direct interpretation of the strong coupling behavior of the decoupled NCOS and NCYM theories.

The structure of the paper is as follows. We begin in Section II by describing the decoupling limit on the M5–brane. In sections III we show that this limit reduces to the NCOS limit on the D4–brane. In Section IV we show that the single limit on the M5–brane reduces to both the NCOS and NCYM limits on the D3–brane. These two limits are related by a modular transformation on the two–torus. Here the role of the open membrane metric is shown to play a crucial role. We end with some conclusions and discussion.

## II. THE M5–BRANE AND THE DECOUPLING LIMIT

The five–brane may be effectively described by a six–dimensional self-dual two form field theory (this is neglecting the superpartners in the (2,0) supermultiplet). The adapted field strength is

$$\mathcal{H} = db + C \ , \tag{4}$$

where $C$ is the pull–back to the five–brane of the three–form potential in eleven–dimensional supergravity and $b$ is the two form potential on the five–brane worldvolume. The self–duality condition provides a nonlinear algebraic constraint involving the components of the field strength and the induced metric $g_{\mu\nu}$ on the brane as follows [10]:

$$\frac{\sqrt{-\det g}}{6}\epsilon_{\mu\nu\rho\sigma\lambda\tau}\mathcal{H}^{\sigma\lambda\tau} = \frac{1+K}{2}(G^{-1})_{\mu}{}^{\lambda}\mathcal{H}_{\nu\rho\lambda}\ , \tag{5}$$

where $\epsilon^{012345} = 1$ and the scalar $K$ and the tensor $G_{\mu\nu}$ are given by

$$K = \sqrt{1+\frac{\ell_p^6}{24}\mathcal{H}^2}\,, \tag{6}$$

$$G_{\mu\nu} = \frac{1+K}{2K}\left(g_{\mu\nu}+\frac{\ell_p^6}{4}\mathcal{H}^2_{\mu\nu}\right)\ , \tag{7}$$

where $\ell_p$ is the 11D Planck scale.

The relation (5) involves the metric, the field strength components and the plank length $\ell_p$. As we want to carry out a scaling in these quanities we must make sure that any scaling obeys the above relation (5). This is amply discussed in [3,9]. To facilitate this we introduce a parametrisation of constant flux solutions to (5) as follows:

$$\mathcal{H}_{\mu\nu\rho} = \frac{h}{\sqrt{1+\ell_p^6 h^2}}\epsilon_{\alpha\beta\gamma}v_\mu^\alpha v_\nu^\beta v_\rho^\gamma + h\,\epsilon_{abc}u_\mu^a u_\nu^b u_\rho^c\ , \tag{8}$$

$$G_{\mu\nu} = \frac{\left(1+\sqrt{1+h^2\ell_p^6}\right)^2}{4}\left(\frac{1}{1+h^2\ell_p^6}\eta_{\alpha\beta}v_\mu^\alpha v_\nu^\beta + \delta_{ab}u_\mu^a u_\nu^b\right)\ . \tag{9}$$

Here $h$ is a real field of dimension $(\text{mass})^3$ and $(v_\mu^\alpha, u_\mu^a)$, $\alpha = 0,1,2$, $a = 3,4,5$, are sechsbein fields in the nine–dimensional coset $SO(5,1)/SO(2,1)\times SO(3)$ satisfying

$$g^{\mu\nu}v_\mu^\alpha v_\nu^\beta = \eta^{\alpha\beta}\ ,\quad g^{\mu\nu}u_\mu^a v_\nu^\beta = 0\ ,\quad g^{\mu\nu}u_\mu^a u_\nu^b = \delta^{ab}\ , \tag{10}$$

$$g_{\mu\nu} = \eta_{\alpha\beta}v_\mu^\alpha v_\nu^\beta + \delta_{ab}u_\mu^a u_\nu^b\,. \tag{11}$$

A derivation of this parametrisation is given in [9].

The relation between the tensor $G_{\mu\nu}$ and the open membrane metric for the five–brane in analogy with the open string metric that occurs on D-branes was discussed in [9]. It should be noted that the overall conformal scale of the open membrane metric was not determined. In this paper such an overall scale will play a role. We therefore define the open membrane metric as follows

$$\hat{G}_{\mu\nu} \equiv \phi(x)\left(g_{\mu\nu} + \frac{\ell_p^6}{4}\mathcal{H}^2_{\mu\nu}\right) , \tag{12}$$

where the function $\phi(x) \neq 0$ and $x$ is given by the dimensionless combination $x = \ell_p^6\mathcal{H}^2$. Using the parametrisation (8), this metric can be written as

$$\hat{G}_{\mu\nu} = (1 + \frac{1}{2}h^2\ell_p^6)\,\phi(h^2\ell_p^6)\left(\frac{1}{1+h^2\ell_p^6}\eta_{\alpha\beta}v_\mu^\alpha v_\nu^\beta + \delta_{ab}u_\mu^a u_\nu^b\right) . \tag{13}$$

Below we shall determine the asymptotic behavior of $\phi(x)$ for large $x$ from the requirements of the decoupling limit.

We now proceed with the definition of the decoupling limit. The properties that we demand for the decoupling limit we wish to take are as follows:

i) The Planck length $\ell_p \rightarrow 0$, so that the gravitational interactions can be decoupled.

ii) The proper six–dimensional lengths $ds^2(\hat{G})$ of the open membrane metric are fixed in eleven–dimensional Planck units in the limit, i.e. $\ell_p^{-2}ds^2(\hat{G})$ is fixed, so that the limit describes a genuine six–dimensional theory with a finite length scale $\ell_g$.

iii) The electric components contain a divergent piece and a constant piece, in analogy with the limit discussed in [4] for open strings.

The first condition we satisfy by scaling $\ell_p \sim \epsilon^{\frac{1}{3}}$ $(\epsilon \rightarrow 0)$. In order to satisfy the second and third condition we impose that $h\ell_p^3$ diverges[3].

We are therefore led to consider the following limit:

---

[3]This is in contrast with the limit in [9] where $h\ell_p^3$ does not diverge.

$$
\begin{aligned}
g_{\alpha\beta} \sim \epsilon^0 \; \Rightarrow v \sim \epsilon^0 \quad ; \quad & g_{ab} \sim \epsilon^1 \Rightarrow u \sim \epsilon^{\frac{1}{2}} \,, \\
\ell_p \sim \epsilon^{\frac{1}{3}} \,, \quad h \sim \epsilon^{-\frac{3}{2}} \,, & \quad \epsilon \to 0 \,.
\end{aligned} \tag{14}
$$

such that the components of $\mathcal{H}$ given by (8) behave as follows:

$$
\begin{aligned}
\mathcal{H}_{012} &\sim \ell_p^{-3}(1 - \frac{1}{2}\ell_p^{-6}h^{-2}) \sim \epsilon^{-1} + \epsilon^0 \,, \\
\mathcal{H}_{345} &\sim hu^3 \sim \epsilon^0 \,.
\end{aligned} \tag{15}
$$

The physics on the five–brane in the decoupling limit is uniquely defined by the two fixed noncommutativity parameters of dimension [length]$^2$ constructed from the the finite parts of $\mathcal{H}_{012}$ and $\mathcal{H}_{345}$ as follows:

$$
\Theta_{\mathrm{T}} \equiv (h^2\, \ell_p^9)^{2/3} \quad , \quad \Theta_{\mathrm{S}} \equiv (h\, u^3)^{-2/3} \,, \tag{16}
$$

where we have set $v = 1$. In order to satisfy requirement (ii) we demand in analogy with [3]

$$
\ell_p^2\, (\hat{G}^{-1})^{\mu\nu} \equiv \ell_g^2\, G_{\mathrm{OM}}^{\mu\nu} \text{ is fixed} \,. \tag{17}
$$

This allows us to fix the conformal factor as follows:

$$
\phi(x) \sim x^{-\frac{2}{3}} \quad \text{as} \quad x \to \infty \;. \tag{18}
$$

With the conformal factor now fixed we find

$$
\ell_p^2\, (\hat{G}^{-1})^{\mu\nu} = (\Theta_{\mathrm{T}}\eta^{\alpha\beta} \oplus \Theta_{\mathrm{S}}\delta^{ab}) \equiv \ell_g^2\, G_{\mathrm{OM}}^{\mu\nu} \;. \tag{19}
$$

This defines a noncommutative M5–brane length scale $\ell_g$, a fixed metric $G_{\mathrm{OM}}^{\mu\nu}$ and a dimensionless parameter $\lambda$ as follows

$$
\ell_g \equiv \sqrt{\Theta_{\mathrm{T}}} \;, \quad \lambda \equiv \frac{\Theta_{\mathrm{S}}}{\Theta_{\mathrm{T}}} \,, \quad G_{\mathrm{OM}}^{\mu\nu} = (\eta^{\alpha\beta} \oplus \lambda\delta^{ab}) \;. \tag{20}
$$

## III. THE NCOS LIMIT ON THE D4–BRANE

In this section we show that the decoupling limit (14) on the M–theory five–brane reduces to the NCOS limit on the D4–brane. This provides an interpretation of the spatio-temporal noncommutative five–brane as the strong coupling dual of the NCOS on the D4–brane.

In order to show this we wrap the five–brane, for finite $\epsilon$, on a circle of fixed radius $R$ in the direction $x^2$ and identify

$$x^2 = X^{11} \sim X^{11} + R\,, \qquad \mathcal{F}_{AB} = R\mathcal{H}_{AB2}, \qquad A, B = 0, 1, 3, 4, 5\,. \tag{21}$$

Clearly this means that only $\mathcal{F}_{01}$ is nonzero on the D4–brane. We also use the following standard relations between M–theory and IIA string theory parameters:

$$g_s = \left(\frac{R}{\ell_p}\right)^{\frac{3}{2}}\,, \qquad \alpha' = \frac{\ell_p^3}{R}\,. \tag{22}$$

The scaling of the metric components in $D = 11$ induces the same scaling for the ten–dimensional metric components and together with the requirement of fixed radius $R$ we find the following limit on the D4–brane (we reset our conventions such that $\alpha, \beta = 0, 1$)

$$\begin{aligned} &g_{\alpha\beta} \sim \epsilon^0\,, \quad g_{ab} \sim \epsilon^1\,, \quad \mathcal{F}_{01} \sim \epsilon^{-1} + \epsilon^0\,, \\ &\alpha' \sim \epsilon^1\,, \quad g_s \sim \epsilon^{-\frac{1}{2}}\,, \quad \epsilon \to 0\,. \end{aligned} \tag{23}$$

As a result we find that length scales on the D4–brane, as measured by the open string metric $G^{AB}$, are kept fixed in the limit. This also holds for the noncommutativity parameters $\Theta^{AB}$ appearing in the two–point function (1) and the open string coupling $G_{\mathrm{O}}$ given by (2). We identify the NCOS limit on the D4–brane with electric field strength $\mathcal{F}_{01} = \mathcal{F}_c - \frac{1}{2}\theta^{-1}$, where the diverging critical electric field $\mathcal{F}_c$ and the fixed noncommutativity parameter $\theta$ are given by

$$\mathcal{F}_c = R\ell_p^{-3}\,, \qquad \theta = \frac{h^2\ell_p^9}{R}\,. \tag{24}$$

Hence, using (15) and (16), we can write fixed D4–brane quantities in terms of the fixed five–brane data $\Theta_{\mathrm{T}}$, $\Theta_{\mathrm{S}}$ and $R$, or equivalently $\ell_g$, $\lambda$ and $R$:

$$\alpha' G^{AB} = (\frac{\Theta_{\rm T}^{\frac{3}{2}}}{R}\eta^{\alpha\beta} \oplus \frac{\Theta_{\rm T}^{\frac{1}{2}}\Theta_{\rm S}}{R}\delta^{ab}) = \frac{\ell_g^3}{R}(\eta^{\alpha\beta} \oplus \lambda\delta^{ab}) = \frac{\ell_g^3}{R}G_{\rm OM}^{AB}\,, \tag{25}$$

$$\theta^{AB} = (\frac{\Theta_T^{\frac{3}{2}}}{R}\epsilon^{\alpha\beta} \oplus 0) = \frac{\ell_g^3}{R}(\epsilon^{\alpha\beta} \oplus 0)\ , \tag{26}$$

$$G_{\rm O} = \left(\frac{R}{\ell_g}\right)^{\frac{3}{2}}\ . \tag{27}$$

Therefore the NCOS limit on the D4–brane has an effective open string scale $\alpha'_{\rm eff}$ and noncommutativity parameter $\theta$ given by

$$\alpha'_{\rm eff} \equiv \theta = \frac{\ell_g^3}{R}\,, \quad G_{\rm OM}^{AB} = G_{\rm OS}^{AB}\,. \tag{28}$$

Thus we find the following relations between open string moduli and M–theory open membrane moduli:

$$R = G_O\sqrt{\alpha'_{\rm eff}} \tag{29}$$

$$l_g = G_O^{\frac{1}{3}}\sqrt{\alpha'_{\rm eff}}\,. \tag{30}$$

These are formally equivalent to the standard relations between the moduli of M-theory and IIA superstring theory provided that we give $\ell_g$ a six–dimensional interpretation analogous to that of the eleven–dimensional Planck scale $\ell_p$ in M–theory. This suggests that the NCOS theory on the D4–brane generates an extra dimension when we increase the open string coupling and in the limit $R \to \infty$ we end up with a noncommutative (in all directions!) six–dimensional theory governed by the scale $\ell_g$, as displayed in Figure 1. Note that $\alpha'_{\rm eff} = \theta$ implies that a field theory limit taking $\alpha'_{\rm eff} \to 0$ will at the same time also result in vanishing spatio-temporal noncommutativity.

FIGURES

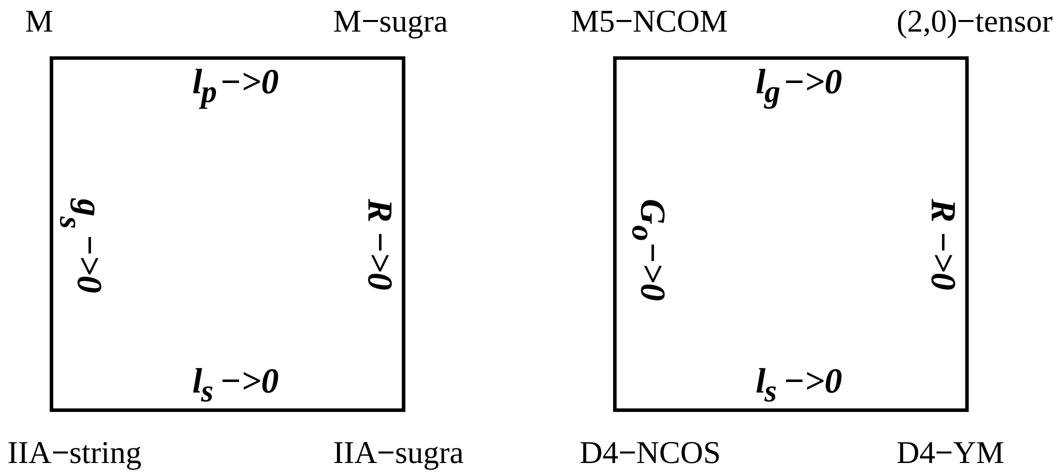


FIG. 1. The square on the left relates M-theory and IIA-superstring theory and their low energy limits. Similarly, the square on the right displays the relations between the D4-brane noncommutative open string theory (NCOS) and its M5-brane noncommutative open membrane (NCOM) origin, and their low energy limits.

## IV. THE NONCOMMUTATIVE LIMITS ON THE D3–BRANE

Next we carry out the double–dimensional reduction of the M–theory five–brane on a fixed two–torus in order to compare with the limits given in [4] for the D3–brane directly reduced on a circle. We drop all Kaluza–Klein modes and identify the wrapped five–brane with the directly reduced D3–brane in nine dimensions. This gives the following relations between M–theory five brane and IIB three brane quantities [12–14]

$$(x^2, x^5) = (X^{11}, X^9) \sim (X^{11} + R_2, X^9 + R_5)\,, \tag{31}$$

$$\mathcal{F}_{AB} = R_2 \mathcal{H}_{AB2}\,, \qquad \frac{g^{(E)}_{AB}}{\alpha'} = \frac{\sqrt{R_2 R_5 u}}{\ell_p^3} g_{AB}\,, \qquad A, B = 0, 1, 3, 4\,, \tag{32}$$

$$g_s = \frac{R_2}{R_5 u}\,, \qquad \frac{R_B^{(S)}}{\sqrt{\alpha'}} = \frac{\ell_p^{\frac{3}{2}}}{R_2^{\frac{1}{2}} R_5 u}\,. \tag{33}$$

where the two–torus coordinate periodicities $R_2$ and $R_5$ are fixed quantities (we set the real part of the complex structure equal to zero) and where $u$ is the scale of the induced dreibein on the five–brane in the 3, 4, 5 directions. The quantity $R_B^{(S)}$ is the IIB radius in string frame and $g_{MN}^{(E)}$ is the IIB Einstein metric which is related to the IIB string metric by $g_{MN}^{(E)} = g_s^{-\frac{1}{2}} g_{MN}^{(S)}$.

The S-dual description of the D3–brane, giving a magnetic field strength, can be obtained by performing a modular transformation on the two–torus, which gives the following relations between the quantities in M–theory and the S-dual picture:

$$\tilde{\mathcal{F}}_{AB} = R_5 \mathcal{H}_{AB5}\,, \qquad \tilde{g}_s = \frac{R_5 u}{R_2}\,, \qquad \frac{\tilde{g}_{AB}^{(E)}}{\tilde{\alpha}'} = \frac{g_{AB}^{(E)}}{\alpha'}\,, \quad \frac{\tilde{R}_B^{(S)}}{\sqrt{\tilde{\alpha}'}} = g_s^{-\frac{1}{2}} \frac{R_B^{(S)}}{\sqrt{\alpha'}}\,, \tag{34}$$

where the tilde denotes quantities in the the IIB S-dual picture.

Inserting the scaling limit for the M5–brane (14) in the first set of relations above we obtain the following scaling limit for the D3–brane:

$$\begin{aligned} &\mathcal{F}_{01} \sim \epsilon^{-1} + \epsilon^0\,, \qquad \mathcal{F}_{34} = 0\,, \\ &\frac{g_{AB}^{(S)}}{\alpha'} \sim \mathrm{diag}(\epsilon^{-1}, \epsilon^{-1}, \epsilon^0, \epsilon^0)\,, \qquad g_s \sim \epsilon^{-\frac{1}{2}}\;. \end{aligned} \tag{35}$$

We identify (35) as the open string limit where the critical field strength and the noncommutativity parameter $\theta_{\rm NCOS}$ are given in terms of M–theory quantities by

$$\mathcal{F}_c = R_2 \ell_p^{-3}\,, \qquad \theta_{\rm NCOS} = \frac{h^2 \ell_p^9}{R_2}\;. \tag{36}$$

Identifying the finite quantities on the D3–brane with the finite quantities on the wrapped M five–brane we obtain the following relations (we reset our conventions such that $\alpha = 0, 1$ and $a = 3, 4$):

$$\alpha' G^{AB} = \Big(\frac{\Theta_{\rm T}^{\frac{3}{2}}}{R_2}\eta^{\alpha\beta} \oplus \frac{\Theta_{\rm T}^{\frac{1}{2}}\Theta_{\rm S}}{R_2}\delta^{ab}\Big) = \frac{\ell_g^3}{R_2}\left(\eta^{\alpha\beta} \oplus \lambda\delta^{ab}\right)\,, \tag{37}$$

$$\Theta^{AB} = \Big(\frac{\Theta_{\rm T}^{\frac{3}{2}}}{R_2}\epsilon^{\alpha\beta} \oplus 0\Big) = \frac{\ell_g^3}{R_2}(\epsilon^{\alpha\beta} \oplus 0)\,, \tag{38}$$

$$G_{\rm O}^2 = \frac{R_2}{R_5}\sqrt{\frac{\Theta_{\rm S}}{\Theta_{\rm T}}} = \frac{R_2}{R_5}\sqrt{\lambda}\,, \tag{39}$$

$$r_{\rm B} \equiv \frac{R_{\rm B}^{(S)}}{\sqrt{\alpha'}} = \frac{\Theta_{\rm S}^{\frac{1}{2}}\Theta_{\rm T}^{\frac{1}{4}}}{R_2^{\frac{1}{2}}R_5} \; . \tag{40}$$

Hence the NCOS limit on the D3–brane has effective open string scale $\alpha'_{\rm eff}$ and noncommutativity parameter $\theta_{\rm NCOS}$ both given by

$$\alpha'_{\rm eff} \equiv \theta_{\rm NCOS} = \frac{\Theta_{\rm T}^{\frac{3}{2}}}{R_2}\,, \tag{41}$$

Importantly, the worldsheet sigma model coupling constant $r_{\rm B}$ given by (40) remains finite in the limit.

In the S-dual/modular transformed description we find (inserting the five brane limit into the second set of relations):

$$\tilde{\mathcal{F}}_{01} = 0\,, \qquad \tilde{\mathcal{F}}_{34} = \epsilon^0 \tag{42}$$

$$\frac{\tilde{g}_{AB}}{\tilde{\alpha}'} \sim \mathrm{diag}(\epsilon^{-\frac{1}{2}}, \epsilon^{-\frac{1}{2}}, \epsilon^{\frac{1}{2}}, \epsilon^{\frac{1}{2}})\,, \qquad \tilde{g}_s \sim \epsilon^{\frac{1}{2}} \tag{43}$$

This we identify with the noncommutative field theory limit as described in [3]. In this case the mass scales on the D3–brane are sent to zero, i.e. $\tilde{\alpha}' G^{AB} \to 0$, decoupling the massive string modes.

The relations between the fixed M–theory quantities and the fixed quantities on the D3–brane are as follows:

$$g^2_{YM} = \frac{R_5}{R_2\sqrt{\lambda}}\;, \qquad \theta_{\rm NCYM} = \frac{\Theta_{\rm S}^{\frac{3}{2}}}{R_5}\;, \tag{44}$$

$$m = \frac{\Theta_{\rm S}^{\frac{1}{4}}}{\sqrt{R_5}R_2}\;, \tag{45}$$

where $m$ is the periodicity in mass units of the compact scalar in the four–dimensional noncommutative action. Note that for the noncommutative field theory, instead of a fixed worldsheet sigma model radius $r_{\rm B}$ we now find a fixed kinetic term for the compact transverse scalar $\Phi \equiv \frac{X^9}{R_B^{(S)}}$ in the limit when $\epsilon \to 0$.

The natural fixed moduli for the noncommutative M-five brane will be $l_g$, the complex structure of the torus, $\tau_{OM}$ and the area of the torus, $A_{OM}$ as measured by the open membrane metric:

$$\tau_{OM} = \frac{R_2}{R_5}\sqrt{\lambda}\,, \quad A_{OM} = R_2 R_5 \frac{1}{\sqrt{\lambda}} \tag{46}$$

We now recover the standard relations between M theory and the S-dual descriptions of IIB for the noncommutative open string/membrane moduli. For the noncommutative open string,

$$\begin{aligned} G_O &= \tau_{OM} \\ r_B &= A_{OM}^{-\frac{3}{4}} \tau_{OM}^{\frac{1}{4}} l_g^{\frac{3}{2}} \,. \end{aligned} \tag{47}$$

For the S-dual, noncommutative field theory,

$$\begin{aligned} g_{YM}^2 &= \frac{1}{\tau_{OM}} \\ m &= A_{OM}^{-\frac{3}{4}} \tau_{OM}^{-\frac{1}{4}} l_g^{\frac{1}{2}} \,. \end{aligned} \tag{48}$$

The duality transformation is now obtained by a modular transformation on the torus as seen by the open membrane metric so that the two theories are related by:

$$\tau_{OM} \to \frac{1}{\tau_{OM}} \tag{49}$$

with the appropriate interpretation of duality related quantities. We remark that the couplings are independent of our choice of conformal factor for the open membrane metric. The duality between NCOS and NCYM is possible for the D3–brane because both open string coupling and Yang-Mills coupling are independent of the closed string scale $\alpha'$. Finally, we consider the following limits of the NCOS on the D3–brane (we set $\lambda = 1$ below):

1) $T$: Taking $r_{\rm B} \to 0$ while keeping $G_{\rm O,A} \equiv r_{\rm B}^{-1} G_{\rm O}$ fixed leads to the NCOS on the T-dual D4–brane with open string coupling $G_{\rm O,A}$. This is analogous to how the usual closed string coupling transforms under T-duality. It is interesting that this noncommutative open string theory exhibits a sort of T-duality.

2) $S$: Taking $G_{\rm O} \to \infty$ while keeping $\theta_{\rm NCYM} \equiv G_{\rm O}\sqrt{\alpha'_{\rm eff}}$ fixed leads to the S-dual NCYM on the D3–brane with $g_{YM} \to 0$, as expected.

## V. DISCUSION

We have argued for the existence of a decoupled noncommutative theory on the five–brane defined by the limit (14) by showing its relation to various well–defined limits of IIA and IIB string theory. Ultimately we are of course interested in finding an intrinsically six–dimensional definition of the decoupled theory. One may wonder to what extent the open membrane action underlies such a formulation and in particular whether there is an analog of the subtle cancellations between the diverging electric field and tension that occur in the string case. In the critical limit we expect the finite parts of the Wess–Zumino term to yield the non–commutative structure of the five–brane loop–space via the definition of the functional Moyal product given in [9]. The role of the kinetic part of the action is more unclear, however, due to the usual membrane instability. Interestingly one may construct a stable, non–degenerate open membrane solution in the critical limit which is *not* a limit of any solution for finite $\epsilon$. This is the analogue of the the critical string solution discussed in [5]. One would hope that the quantisation of the membrane in this near critical background will provide the open membrane metric with the appropriate conformal factor given in this paper. This is ongoing work. It is not yet clear whether the critical field will cure the usual membrane sicknesses.

Finally we wish to make the following observation, given our choice of conformal factor for the open membrane metric we see that the line element for the self–dual string solution [15] is exactly $AdS_3 \times S^3$ in the near horizon limit. Given that the proceedure in this paper has been to reproduce the usual bulk relations for the decoupled theories on the brane one wonders whether it might be possible to formulate an AdS/CFT corresspondence [16] for the self–dual string in the five brane.

In summary, the NCOS on the D4–brane with noncommutativity parameter $\theta = \alpha'_{\text{eff}}$ has a dual description in the limit of strong coupling $G_O >> 1$ as a noncommutative five–brane with fundamental length $\ell_g = G_O^{\frac{1}{3}}\sqrt{\alpha'_{\text{eff}}}$ reduced on a circle of radius $R = G_O\sqrt{\alpha'_{\text{eff}}}$. The S–duality of the NCOS and NCYM theories on a directly reduced D3–brane follows from the modular invariance of the noncommutative five–brane wrapped on a torus. The couplings on the D3–brane are identified with the complex structure of the torus in the open membrane metric.

## Note Added

During the completion of this paper we received the preprint [11] that also discusses the noncommutative open membrane limit and its relation to the noncommutative open string limit on the D4–brane. The preprint [11] also contains an interesting discussion of NCOS theories at strong coupling for the other D–branes. Our paper emphasizes the role played by the open membrane metric and the relation between M/IIB moduli.


## Acknowledgements

P.S. is grateful to M. Cederwall, Henrik and B.E.W. Nilsson for discussions. D.S.B. thanks R. Gopakumar for discussions during the Fradkin Memorial conference on *Quantization, Gauge Theory and Strings*. This work is supported by the European Commission TMR program ERBFMRX-CT96-0045, in which E.B., D.S.B. and J.P.v.d.S. are associated to the University of Utrecht. The work of J.P.v.d.S. and P.S. is part of the research program of the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM).

## REFERENCES


[1] A. Connes, M. R. Douglas and A. Schwarz, "Noncommutative geometry and matrix theory: Compactification on tori," JHEP **9802** (1998) 003 [hep-th/9711162];

M. R. Douglas and C. Hull, "D-branes and the noncommutative torus," JHEP **9802** (1998) 008 [hep-th/9711165];

Y. E. Cheung and M. Krogh, "Noncommutative geometry from 0-branes in a background B-field," Nucl. Phys. **B528** (1998) 185 [hep-th/9803031].

[2] A. Schwarz, "Morita equivalence and duality," Nucl. Phys. **B534** (1998) 720 [hep-th/9805034];

C. Hofman and E. Verlinde, "U-duality of Born-Infeld on the noncommutative two-torus," JHEP **9812** (1998) 010 [hep-th/9810116].

[3] N. Seiberg and E. Witten, "String theory and noncommutative geometry," JHEP **9909** (1999) 032, [hep-th/9908142].

[4] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, "S-duality and Noncommutative Gauge Theory", [hep-th/0005048].

[5] N. Seiberg, L. Susskind and N. Toumbas, "Strings in Background Electric Field, Space/Time Noncommutativity and A New Noncritical String Theory", [hep-th/0005040].

[6] J. L. Barbon and E. Rabinovici, "Stringy fuzziness as the custodian of time-space noncommutativity," hep-th/0005073.

[7] G. Chen and Y. Wu, "Comments on noncommutative open string theory: V-duality and holography," hep-th/0006013.

[8] I. R. Klebanov and J. Maldacena, "1+1 Dimensional NCOS and its U(N) Gauge Theory Dual," hep-th/0006085.

[9] E. Bergshoeff, D. S. Berman, J. P. van der Schaar and P. Sundell, "A Noncommutative

M-theory Five-brane", [hep-th/0005026].

[10] P. S. Howe and E. Sezgin, "Superbranes," Phys. Lett. **B390** (1997) 133 [hep-th/9607227];

P. S. Howe and E. Sezgin, "D = 11, p = 5," Phys. Lett. **B394** (1997) 62 [hep-th/9611008];

P. S. Howe, E. Sezgin and P. C. West, "Covariant field equations of the M-theory five-brane," Phys. Lett. **B399** (1997) 49 [hep-th/9702008].

E. Sezgin and P. Sundell, "Aspects of the M5-brane," hep-th/9902171, proceedings of the Trieste Conference on Superfivebranes and Physics in 5+1 Dimensions, 1-3 April, 1998, Trieste, Italy.

[11] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, "OM Theory in Diverse Dimensions", [hep-th/0006062].

[12] J. H. Schwarz, "An SL(2,Z) multiplet of type IIB superstrings," Phys. Lett. **B360** (1995) 13 [hep-th/9508143].

[13] P. S. Aspinwall, "Some relationships between dualities in string theory," Nucl. Phys. Proc. Suppl. **46** (1996) 30 [hep-th/9508154].

[14] D. S. Berman, "M5 on a torus and the three brane" Nucl. Phys. **B533** (1998) 317 [hep-th/9804115].

[15] P. S. Howe, N. D. Lambert and P. C. West, "The self-dual string soliton," Nucl. Phys. **B515**, 203 (1998) [hep-th/9709014].

[16] J. Maldacena, "The large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2** (1998) 231 [hep-th/9711200].